10 No 4 (New Series No 16) March 1950



# ANALYSIS





CONTENTS



TWO SHILLINGS NET
Annual subscription 10s. 6d. post free for six numbers

ASIL BLACKWELL · BROAD STREET · OXFORD

ANALYSIS 10 . 4 MARCH 1950

# MULTIPLY GENERAL SENTENCES

*By* STUART HAMPSHIRE

THERE is a logically peculiar class of sentences, some members of which have played an important part in philosophical arguments, without their logical peculiarities being generally noticed and labelled; my purpose is simply to indicate in what respect they are logically peculiar, and to attach a label to them; unless their peculiarity is recognised and labelled, they are always liable to mislead. I cannot define this class of sentences, which I shall call 'multiply general sentences', in the sense of providing formal or syntactical rules for recognising any sentence as being a member of the class, since their logical peculiarity is not constantly reflected in their form or syntax; precisely this makes them a philosophical problem worth investigating.

1. Consider two sentences which might be used as primitive formulations of the uniformity of Nature.

(*a*) Every event has a cause.

(*b*) Every event is an instance of some natural law.

Both these sentences look like general statements which can be confirmed or falsified by the methods which logic books prescribe as appropriate to the confirmation or falsification of statements of the form "All X's are Y's"; and they have often in fact been accepted as true or as probably true, or rejected as false or probably false, on the assumption that the normal recognised methods of confirming or falsifying sentences of the form "All X's are Y's" are in principle applicable to them; and many of those who have accepted these sentences as true, or probably true, statements, would not have so accepted them, had they reflected further on the method of their confirmation or falsification, and not simply assumed that they are in this respect indistinguishable from other sentences of the same grammatical form. Of course many, and perhaps most, users of these sentences have for a variety of reasons not intended them to be taken as confirmable or falsifiable general statements, or as empirical statements of any kind; but some certainly have, and even those who have not, have not generally recognised their peculiarity as multiply general sentences among their other peculiarities.[1]

[1] That they contain only semi-formal or logical expressions—'cause', 'event', 'natural law', etc.—is another, more widely recognised and (I think) independent peculiarity of these sentences, distinguishing them from the standard cases of empirical general statements.

2. By a multiply general sentence I mean a sentence beginning with some sign of unrestricted[1] generality — ' all ', ' every ', ' no ', ' nothing ', ' nobody ', etc.—which is so constructed that no singular statement can be formulated which entails or is incompatible with it ; by ' singular statement ' I here mean any statement which (*a*) does not contain any sign of unrestricted generality and (*b*) does contain either a demonstrative expression (*e.g.* ' this ', ' that ', ' there ', ' now ' or a personal pronoun) or a proper name. It is a characteristic (but not a sufficient defining characteristic) of a multiply general sentence that, in addition to the initial sign of generality, it will contain some second sign of unrestricted generality and at least one sign of negation. The effect of such combinations within a single sentence of two signs of unrestricted generality and a sign of negation is to preclude the construction of any singular statement which entails either the truth or falsity of the multiply general sentence.

Consider as a second example the multiply general sentence, "All men are mortal ", so often wrongly cited in logic-books as a typical general statement, and so often accepted as a true empirical statement, on the grounds that no negative instance which refutes it has in fact been discovered. How in principle could the truth or falsity of this dictum be established ? What singular sentence (*i.e.* sentence not itself containing any sign of unrestricted generality) can be constructed which would entail either that it is true or that it is false ? The answer is that no such singular statement can in principle be constructed, and that this dictum cannot in principle be either verified or falsified in the sense in which any general statement not of this peculiar pattern can be either verified or falsified. Those who accept this sentence as a true statement, on the grounds that no negative instance refuting it has in fact been discovered, are in error, the victims of a logical confusion ; for it is *logically* precluded that any such a negative instance should ever be found, whatever the nature of our experience or the longevity of men. On the other hand the sentence " all men are immortal " is a general statement which can in principle be falsified, in the sense that we can construct a singular sentence, *i.e.* " this man has died ", which is logically incompatible with it ; " all men are immortal " is therefore a general statement of the normal recognised pattern—' normal ' in the sense that, presented with any sentence of the form "All X's are Y's ", we tend always to assume that a negative instance is in principle discoverable. I am arguing only that this assump-

[1] The sign of generality is unrestricted when it is not used to indicate a closed or finite class of things, persons or events.

tion, encouraged by text-books of logic, is unsound and needs to be restricted.

But I cannot state any general and formal restriction which would enable one infallibly to distinguish the normal from the abnormal cases; and I am inclined to believe that no such general and formal restriction can in principle be stated. One reason is that the multiple generality is not sufficiently disclosed in the form or grammar of these sentences, but emerges only from considering the use of the particular descriptive words involved. The negative instance which would falsify "All men are mortal" cannot be constructed, because, following the ordinary rules of syntax, we obtain as the negative instance the sentence "this man is immortal"; but to say of a man that he is immortal is to say that he will never (unrestrictedly) die, and this is a general statement and not a singular statement in the sense required. As "This man is immortal" is itself a general statement, and is therefore falsifiable but not conclusively verifiable, it follows that "All men are mortal" is not in principle conclusively falsifiable by the discovery of a negative instance.

What is philosophically interesting is that most people, if asked, would probably say that they believe that it is true that all men are mortal, and would justify the assertion by appeal to observation and experience; they would take themselves to be holding a belief about a matter of fact and to be making an empirical statement. But they would not be able to say, if asked, what it would be like to observe or recognise an immortal man; they would admit, if challenged, that they had not indicated what possible experience they were excluding when they claimed to believe that all men are mortal. Probably many people, when so challenged, would decide that the belief which they are expressing when they say "all men are mortal" is more clearly and less misleadingly expressed as "all men decay" or "all men are senescent"; that is, they would agree that they would withdraw their statement, and abandon their belief, if and perhaps only if, they discovered a man who showed no signs of senescence or decay; they might *decide* that this is the possible observation which (perhaps without realising it) they were excluding. Compare here the multiply general sentence "There are no perpetual motion machines", which probably means what would be more clearly expressed by the sentence "All machine dissipate energy"; more clearly expressed, if, when I assert that there are no perpetual motion machines, the only possibility which I intend to exclude is the observation of a machine which does not dissipate energy. But I might not have realised, or made clear

and explicit to myself, that this is the possibility which I intend to exclude, until I am challenged. In this legitimate and important sense I may often not know what (if anything) I am asserting when I use a multiply general sentence ; and thus am likely to mislead others, who, in order to understand me, must infer from the context what possibility I am excluding, and therefore what I am asserting. In this artificially restricted, but now familiar and useful sense, multiply general sentences may legitimately be described as meaningless or as not genuine empirical statements ; ' meaningless ', not implying that they are not often intelligently and intelligibly used, but that they are always inexplicit and to be replaced by a sentence showing explicitly what definite assertion (if any) is actually intended.

3. Each of the uniformity of Nature sentences has the same quasi-syntactical peculiarity as "All men are mortal" ; no singular statement (*i.e.* statement containing no sign of unrestricted generality) can in principle be constructed which either entails or is incompatible with either of them. Each of them involves the double occurrence of the sign of unrestricted generality, the second being implicit or half-concealed in the words ' cause ', ' law ', and ' predictable '. But the involutions of generality are here even more complicated. For the putative negative instance " This is an uncaused event ", which entails "There is *no* other event which *never* occurs without this event occurring ", is itself a multiply general sentence, for which in turn no negative instance can be constructed; so if "All men are mortal" is a doubly general sentence, " Every event has a cause " is trebly general.

Multiply general sentences occur in several other metaphysical theses or arguments,[1] *i.e.* in theses or arguments which look as if they were empirical or factual, but, when probed, turn out not to be. The trick or snare of the unrecognised multiply general sentence is a trick of *language*, and depends on the interpretation given to particular expressions (not purely logical or syntactical expressions) in particular contexts ; and it is for this reason that I have not appealed to any logicians' symbolism, in which such sentences or formulae either are, or can easily be, excluded.

*University College, London.*

[1] The assertion or denial of a First Cause or First Event is another example of an old metaphysical dispute, which involves the use of a multiply general sentence.

# HETEROLOGY AND HIERARCHY

*By* NATHANIEL LAWRENCE

## I

IN *An Inquiry into Meaning and Truth,* Bertrand Russell states that "The arguments for the necessity of a hierarchy of languages are overwhelming, and I shall henceforth assume their validity."[1] A score of pages later, however, he cannot resist the opportunity to demonstrate by exhibiting an 'antinomy', that "the hierarchy . . . is essential".[2] The antinomy is one associated with the word 'heterological'.

The object of the present paper is to examine this alleged antinomy to discover whether or not it supports the necessity for a hierarchy of languages. The general question of whether there is *any* need for a hierarchy of languages is one which could stand considerable critical appraisal. At present there is a runaway enthusiasm among philosophical students of language for an affirmative answer to this question.[3] But the general problem is not here examined. I merely consider the smaller problem: Does the paradox connected with the term 'heterological' require capitulation to a theory of language hierarchy?

## II

I shall first consider Russell's statement of the paradox, expanding the latter part of it for the sake of clarity.

> "A predicate is 'heterological' when it cannot be predicated of itself; thus 'long' is heterological because it is not a long word, but 'short' is homological. ['German', 'learned', 'beautiful' are heterological; 'English', 'erudite', 'ugly' are homological].[4] We now ask: Is 'heterological' heterological? Either answer leads to a contradiction. To avoid such antinomies, the hierarchy of languages is essential."[5]

The 'contradiction' to which Russell refers needs examination. Clearly what Russell means is that should we answer either 'yes' or 'no' to the question, "Is 'heterological' heterological?" we obtain results which issue in self-contradictory conclusions. Let us consider the two cases.

(1) If I say, "Yes, 'heterological' is heterological", then I am saying that it is incapable of self-predication; but the first

[1] W. W. Norton, New York, 1940, p. 75.

[2] *Op. cit.*, p. 97.

[3] By no coincidence at all, there is, among this same group of people, a careless enthusiasm for the proposition that Plato regarded the 'forms' as disparate entities residing in 'heaven'. *E.g.*, Russell, *op. cit.*, p. 27.

[4] The sentence in square brackets is Russell's footnote to the passage.

[5] *Op. cit.*, pp. 96–97.

part of my argument consists in doing just that. From the assertion in which I predicate a term of itself, I draw the conclusion that such predication is impossible. So far, so good. Let us, for the time being, assume that this part of the antinomy holds.

(2) If I say, "No, 'heterological' is not heterological", then I am saying that it *is* capable of self-predication; however, the first part of my assertion is one in which I specifically *deny* the predicate of heterologicality to 'heterological'. So once again I have apparently made an assertion from which a conclusion is drawn that contradicts the assertion.

Before I examine the 'antinomy' further I shall first consider two features of the property of being heterological with which Russell does not deal. These two points will serve to sharpen the evident need for a careful examination of the meaning of heterologicality.

(i) If it is a paradox that is desired we do not need to go so far as to ask whether or not 'heterological' is self-predicable. There are paradoxes aplenty without involving ourselves in a regress of languages. For instance, 'heterological' and 'homological' must be contradictories or the paradox we have examined falls through. Now we ask, "Is 'long' heterological?" The answer is that it is. 'Long' is a short word. Then if anything is homological (non-heterological) it must be non-long. But 'polysyllabic' is homological and is therefore non-long. However, this is false, since the average number of letters in English discourse is five.[1]

(ii) What of the examples Russell gives us? 'Ugly' is judged homological because it does not strike Russell's eye or ear well. 'Erudite' is homological because of the relatively small numbers of people who can define it accurately. But suppose I think 'ugly' is beautiful? It has a nice balance of vertical strokes above and below the scanning line. The closed loop of the *g* is metamorphosed into the generously welcoming open loop of the *y*, and so forth. The whole word has a sense of strength, grace, balance, and friendliness. As for the sound of the word, it is reminiscent of that tenderest of scenes, a child nursing. And so, tongue in cheek or not, I might defend any word. Again, with no change in meaning, the word 'erudite' could become as unerudite as the word 'integrate', which was at one time in the vocabulary of only a few. If Churchill's next speech includes the phrase, "erudite authors of labour policy" the illiteracy with

[1] Space estimation of manuscripts to be set up in type relies upon this assumption, for instance.

regard to the meaning of this word will vanish overnight, with the assistance of the Press.

The first of these two points shows that the concept of heterologicality encounters difficulties before it is ever, so to speak, turned on itself. The second point shows that despite the ideal of a language in which words have, in some sense, a stable status, certain aspect of words will always be fluctuating and open to subjective variability. This second point thus serves to warn us that when we deal with the concept ' heterological ' we are dealing with a characteristic of words which may depend as much upon taste and circumstance as upon logic. I may not turn aside here to the several opportunities for analysis which the presence of these subjectively and otherwise variable factors offers. The two points together should show that when one is asked, " Is ' heterological ' heterological ? " no answer need be given until the notion of heterologicality is further analyzed. It may be that then the question can be shown to be meaningless, for in the definition of a heterological word as being one which is not predicable of itself there is an epidemic ambiguity which is present throughout the definition. There is ambiguity in what is meant by ' word ', by ' heterological ', by ' predicable ', and by ' self '. At bottom these are all the same ambiguity. I shall approach this ambiguity at its most accessible and familiar portal, the variety of meanings of the term ' word '.

## III

The main line of the following analysis is not to be understood as in any sense novel. Russell himself, of course, spends considerable effort in analyzing what is meant by ' word '.[1] and in so doing develops the analytic machinery which, with very little adapting, could have been employed to resolve the paradox of ' heterology ' without introducing the notion of a language hierarchy and the difficulties which are attendant on this notion.

The word ' word ' is, of course, ambiguous. I take some examples, beginning with those which Russell himself chooses to exhibit the puzzle.

(1) The word ' long ' is heterological because it is a short word. Here the word ' long ' apparently refers to the physical properties displayed by a physical object, spoken or written. But it is important to notice that there are at least two meanings of ' word ' buried in this one example. (i) ' Word ' means " unique physical object, numerically different from all similar

[1] *Op. cit.*, ch. 1.

instances ". (ii) ' Word ' also means " that of which this physical instance is an occurrence ". Let us call these $W_1$ and $W_3$ respectively. Each $W_3$ may be either spoken or written. Thus $W_3$ is what is representable by an indefinitely large variety of type faces, etc., on the one hand, and accents, etc., on the other. Each of these indefinitely large groups is distinct from $W_1$ and $W_3$. We rarely call them words ; rather we say they are ways of speaking or kinds of writing or printing, etc. But since they mediate between $W_3$ and $W_1$, let us call them $W_{2s}$ and $W_{2p}$ respectively. Each member of $W_{2s}$ and $W_{2p}$ can itself have an indefinitely large number of $W_1$'s as well as an indefinitely large number of companions in the same group. Thus any $W_1$ is an instance of some $W_2$, which is itself one of many $W_2$'s which are channels of expression for $W_3$. $W_3$ is what we are talking about when we say, " I'm going to find out what that word I heard means ". In these circumstances, we don't care which $W_1$ we get or what $W_1$ it is an instance of although, for psychological reasons, we are likely to prefer some $W_{2p}$ that is legible and a $W_1$ that is also legible.

These psychological reasons are in themselves oblique to the purpose of our present inquiry, but they uncover a pertinent point. We may have a ' legible ' $W_2$ or a ' legible ' $W_1$. Thus we may have a perfectly legible type face of adequate size, but its individual instance may be ' illegible ', being inadequate in ink, on poor paper, or simply too old. Keeping this distinction in mind, what do we mean by saying that a word is ' long ' ? It should be clear at once that we are not talking about $W_1$, although $W_1$ may give us tangible support for our decision. Thus the word ' war ' is longer in most headlines than the word ' inhuman ' is in the account. Again, a little girl of two and a half years, whom I know, repays her parents' prohibitions of her impulses, with interest, by saying, ' No-o-o-o-o-o '. What is required of the judgment that a word is long is that it be referred to other ' words ' of the same $W_2$ group, that is, the same mode of expression of $W_3$. This can be done by selecting other $W_1$'s representing the same $W_2$ group, but there is no need to do so. It is true that I may need a visual or auditory image of other $W_1$'s in order to make any judgment about the length of the word which I am examining, but the function of such an image is analogous to the service which a diagram in a geometry book provides for the student of geometry. It suggests by example. Furthermore, consider the sentence, " Schultz ate a banana ". In $W_{2p}$ the subject of the sentence is longer than the object ; in $W_{2s}$ it is shorter. The length of a word depends

upon the $W_2$ group to which it belongs. That is, when we speak of a word's length we are talking about its $W_2$ properties. Are there any instances where the concept of 'heterological' could be applied to $W_1$'s ? There are ; the words 'black' and 'loud' serve as illustrations for $W_{2p}$ and $W_{2s}$ respectively'[1] And this in turn makes it clear that $W_1$ should also be divided into $W_{1p}$ and $W_{1s}$, with $p$ and $s$ being derivative properties of any $W_1$ gained from the $W_2$ group of which it is an instance.

Let us now consider the homologicality of the word 'erudite'. $W_1$ is obviously out of the picture, except as the immediate mechanical agency of communication. $W_2$ is equally out of the picture, for it does not matter whether the group in which 'erudite' is found is an $s$ group or a $p$ group, or whether it be in a Lancashire burr or in a Limehouse chatter, in brevier extended or pica condensed. It is still an erudite word. On the other hand, it is not as a meaning that 'erudite' is erudite ; since to the extent that it, as a meaning, can be reduced to familiar synonym or simplified explanation, it is available to those who are bookish or not, learned or otherwise. Thus, when we say that the meaning of 'erudite' is itself erudite we are saying somewhat elliptically that to state what 'erudite' means by the use of that juxtaposition of letters is to state the meaning eruditely. It could be stated by other patterns of letters in a way that could be generally understood.[2] When we deal with such words as 'erudite', 'declinable', etc., we are dealing at the $W_3$ level. Here we are dealing with 'word' in the sense that 'erudite' is an English 'word', the sense in which the English language is composed of a certain number of 'words', say a hundred thousand. Thus 'erudite', 'erudit', and 'kenntnisreich' would fall into classifications $W_{3e}$, $W_{3f}$, and $W_{3g}$, symbols for English, French, and German 'words' respectively, and would all be homological. A corresponding listing of 'learned', 'savant', and 'gelehrt' would fall into the same categories respectively, but each would be heterological. Nevertheless, all six 'words' would have the same meanings.[3]

We may safely assume at least one more level of what is meant by a word, then ; this is the controversial level of its meaning, what it seeks to convey, describe, indicate, name, etc.

[1] That is, 'black' is either heterological or not, depending on whether or not black ink is used, etc. ; 'loud' is heterological or not, depending on the vocal force of its instance.

[2] It is, it would seem, dubious whether any meaning is itself erudite, strictly speaking. This problem in reduction of meaning and analysis of definition is one of the many which the present investigation must ignore.

[3] A well-informed analysis of the meanings would undoubtedly show subtle shades of meaning. The difficulty about ideal identity of meanings, that is, exact synonymity, should not damage the significance of the illustration.

Translation is based on the assumption that community of meaning underlies diversity of languages. This assumption flourishes with such words as ' Buch ' and founders with such words as ' Begriff ', but such facts should make us all the more aware of the distinction between $W_3$ and what we shall call $W_4$.

There is no place in this study for a detailed examination of ' meaning ', and no need of one. The distinguishing of $W_1$, $W_2$, $W_3$, and $W_4$ which has been undertaken in these remarks will survive, I think, any of the varieties of analysis to which $W_4$ has been submitted, whether it be the simple division of $W_4$ into denotative and connotative, or one of the more complicated modern analyses '.[1] It is equally possible that a more thorough-going study will reveal other W's ; the division into four types is not presumed to be exhaustive, and any such additions, based on subtler distinctions would reinforce rather than vitiate the conclusions which we are now in a position to draw.

## IV

We have seen that ' heterological ' is a term applied to words which are not predicable of themselves. We have further seen evidence that the expression ' themselves ' in this definition is either elliptical or ambiguous, however, since the ' self ' is not a self at all. The ' word ' which is predicated is always $W_4$. *But that of which it is predicated is not.* Thus when we ask, " Is ' ugly ' ugly ? " we are asking, ' Is the meaning of the word ' ugly ' appropriate to some (customary) symbolic representation of that meaning ? "[2] In general when we ask, " Is ' X ' X ? " we are asking, " Is the meaning of ' X ' appropriate to some level of symbolization of that meaning which is identified by the same name ? "

A glance at our examples shows us that this is the presumption upon which the judgment of the heterologicality of a word rests. ' Long ' is heterological because its $W_4$ is inappropriate (under standardized conditions) to its $W_2$. ' Erudite ' is not heterological, because its $W_4$ is appropriate to its $W_3$. An example of a type not given by Russell will complete the analysis as far as we have taken it. ' Red ' is heterological in the present instance of writing, *i.e.* its $W_4$ is not appropriate to its $W_1$.

[1] *E.g.*, that of C. I. Lewis in *An Analysis of Knowledge and Evaluation*. Open Court, 19 ch. 3.

[2] Actually not all sentences in which the form " Is ' X ' X ? " appears can be regarded as ones in which the second ' X ' (without quotation marks) refers to the meaning of X: *e.g.*, " In French is ' defiance ' defiance ? " But such exceptions do not obscure the analysis of heterologicality.

Were this journal printed in red ink, ' red ' would not be heterological. It is important to recall that this analysis may very well be incomplete. More levels of W's may be required in order to cover adequately the wide variety of meanings possible for ' word '. The present analysis has been undertaken with only certain very limited objectives.

Now let us ask ourselves what has occurred in the case of the question, " Is ' heterological ' heterological ? " Plainly, the second use of the term ' heterological ' in the sentence is designed to point to its $W_4$, its meaning. What about the first appearance of ' heterological ' ? To which kind of W does it direct our attention ? Not merely to itself, *i.e.* its $W_1$, obviously, nor to its $W_2$. What about $W_3$ ? That is, what about regarding the first word as a ' word ' in the sense that any language has a vocabulary of a certain number of ' words ' ? Let us alter $W_3$ and see. Suppose we tentatively add to the Esperanto vocabulary by adding to it the word ' suononpredicato ' as an exact translation of ' heterological '. Now, suppose I ask, ' Is " suononpredicato " heterological ' ? The paradox appears as before. It should be apparent that any synonym offered for ' heterological ' and substituted for that term in its first use in the paradox fails to reduce the paradox. The paradox does not depend upon a consideration of the first appearance of ' heterological ' as a physical object. Nor does it depend upon ' heterological ' as being an example of a mode of writing or speech. And we have just seen that it does not depend upon ' heterological ' as being a word in a language. Rather it depends upon heterological *as having a certain meaning ;* that is, it depends upon the *first* use of ' heterological ' as *also* referring to its $W_4$. It is ' heterological ' as a meaning, not ' heterological ' as an object, a class of expressions, or a constituent element in a language that gets us in trouble. These are all present also but they create no confusion. Our analysis of the meaning of ' heterological ' has shown, however, that the concepts of heterologicality and homologicality depend upon a predication of the meaning of a term of something which is *not* its meaning. That such is the case is clearly indicated by the examples given.

The proper answer, then, to the question, " Is ' heterological ' heterological ? " is that it is non-heterological. But this does not mean therefore that it is homological. One might just as well decide that because a rock is non-ambidextrous it must favour one arm or another. ' Heterological ' is both non-heterological and non-homological. ' Heterological ' and ' homological ' are contradictories only within a universe of discourse which is

confined to words whose $W_4$'s are predicated of W's of some other levels. What is required of a solution of ' the paradox of heterology ' is simply that ' heterological ' be used in an unambiguous and consistent fashion. If this is done, the definition of ' heterological ' which is implicit in the use of the given examples to illustrate the meaning of ' heterological ' prohibits us from attributing either heterologicality or homologicality of it. This prohibition is a warning not to commit a fallacy. Viewed in one light the fallacy is that of equivocation ; viewed in another it is the fallacy of false dichotomy. Other names could perhaps be suitably employed, but that is not important. The fallacy is a familiar one, capable of being exposed by ordinary methods of analysis. There is required no hierarchy of languages, nor even a hierarchy of words (which would by no means necessitate a hierarchy of languages). The claim that the paradox of heterologicality requires the introduction of special techniques and instruments of analysis is unjustified.

*University of Illinois, U.S.A.*

## RUSSELL'S THEORY OF DESCRIPTIONS

*By* P. T. Geach

A RECENT article in Analysis[1] discussed certain *prima facie* difficulties of Russell's theory of definite descriptions. I shall here maintain : (I) that even as applied to his own sort of examples from ordinary language, his analysis of sentences containing definite descriptions is very defective ; (II) that as a convention for a symbolic language his theory involves intolerable complications.

(I) On Russell's view " the King of France is bald " is a false assertion. This view seems to me to commit the fallacy of ' many questions '. To see how this is so, let us take a typical example of the fallacy : the demand for " a plain answer—yes or no !" to the question " have you been happier since your wife died ?" Three questions are here involved :

1. Have you ever had a wife ?
2. Is she dead ?
3. Have you been happier since then ?

[1] Sören Halldén, " Certain Problems connected with the Definition of Identity and Definite Descriptions given in *Principia Mathematica* ", Analysis, 9.2, December 1948.

The act of asking question 2 presupposes an affirmative answer to question 1; if the true answer to 1 is negative, question 2 *does not arise*. The act of asking question 3 presupposes an affirmative answer to question 2; if question 2 does not arise, or if the true answer to it is negative, question 3 *does not arise*. When a question does not arise, the only proper way of answering it is to say so and explain the reason; the 'plain' affirmative or negative answer, though grammatically possible, is *out of place*. (I do not call it 'meaningless' because the word is a mere catchword nowadays). This does not go against the laws of contradiction and excluded middle; what these laws tell us is that *if* the question arose "yes" and "no" *would be* exclusive alternatives.

Similarly, the question "Is the present King of France bald?" involves two other questions:

4. Is anybody at the moment a King of France?

5. Are there at the moment different people each of whom is a King of France?

And it does not arise unless the answer to 4 is affirmative and the answer to 5 negative. (The mere use of the word "King" does not require a negative answer to 5; there used to be two Kings of Sparta at a time). If either of those answers is false, the affirmative answer "yes, he is bald" is not false but simply out of place.[1] This view agrees, I think, with common sense; a plain man, if pressed for an answer, would be very likely to reply: "Don't be silly; there isn't a King of France".

The mere use of the definite description "the King of France" does not *always* presuppose an affirmative answer to question 4 and a negative answer to question 5. For, as Russell rightly holds, these two answers together are logically equivalent to "the present King of France exists"; so, if they were *always* presupposed when we say "the King of France", we could not ask the question whether there is a somebody who "is the King of France" without presupposing that in fact there *is* somebody; which is absurd. Russell's analysis of ordinary *existential* assertions containing definite descriptions, like "the King of France exists", is quite adequate. And again, his analysis works out all right when "the King of France" is a logical *predicate*; "George VI is not the King of France" is logically equivalent to "either George VI is not *a* King of France, or there are several people each of whom is *a* King of France", and it does not assert or presuppose that there is a present King of France. (In the existential assertion itself, it is "the King of France",

[1] *Cf.* Frege, *Die Grundlagen der Arithmetik*, footnote on pp. 87–8.

*not* " exists ", that is the logical predicate ; for the assertion is logically equivalent to " somebody is the King of France ".) But Russell's theory breaks down for sentences in which " the King of France " is a logical *subject*, such as " the King of France is bald " and " an assassin has stabbed the King of France ". Such sentences presuppose an affirmative answer to the question " does the present King of France exist ? " ; since that answer is false, use of " the King of France " as a logical subject is out of place.

It is important to distinguish my view that the existence of the present King of France is *presupposed* by the assertion " the King of France is bald " from Russell's view that his existence is *implied* by this assertion. If $p$ implies $q$, and $q$ is false, $p$ is of course false. But to say $p$ presupposes $q$ is to say that $p$ is an answer to a question that does not arise unless $q$ is true. If $q$ is false, or if $q$ in turn is an answer to a question that does not arise, the assertion of $p$ is not false but simply out of place.

(II) The incorrectness of Russell's theory as an account of ordinary language in no way goes against it as a proposed convention of symbolism. In symbolic language we try to avoid the situations in which a formula is " out of place " because it is an answer to a question that does not arise ; we do this by altering the meaning of the question. Take the case of zero. In ordinary language, the question " May I give you some more tea ? " presupposes that you have already had some ; otherwise it is out of place—as Alice said : " I've had nothing yet, so I can't take more ". The Mad Hatter's retort was : " You mean you can't take *less* ; it's very easy to take *more* than nothing ". The Hatter, like mathematicians, treats " nothing " or " 0 " as an answer to the question " how much ? " on the same level as any other answer ; this does not quite fit ordinary usage, but symbolically it is most convenient.

There are, however, decisive technical reasons against Russell's theory. Russell does not define the definite description '$(\iota x)$ $(Fx)$' as such, but only its use in a context '$\iota x$' ; '$G \ldots (\iota x)(Fx)$' is defined to mean '$(Ey) : Gy . (x) . Fx \equiv x = y$'. Now in applying this definition we have to decide what is the context represented by '$G \ldots$'. For instance, if we apply the definition to a formula containing '$(\iota x)$ $(Fx)$' in one of its clauses, we have to decide whether '$G(\iota x)(Fx)$' is to be taken as short for this clause or for the whole sentence ; and the results of expounding the formula will be different in the two cases. To avoid different *definientia* for the same *definiendum*, Russell lays down rules in *Principia Mathematica* (*14) as to the " scope "

of a definite description—i.e. rules to determine how much of a formula is to be taken as the '$G(\iota x)(Fx)$' of the definition. Unfortunately, these rules are insufficient. Take the expression '$(\iota x)(Fx)\ R\ (\iota x)(Fx)$'. This contains no part that is a sentence or sentential function in which '$(\iota x)(Fx)$' occurs; hence Russell's conventions of "scope" are inapplicable. But there are several different ways of expounding it according to the above definition.

(i) One might take '$G$' in the *definiendum* to represent '$\ldots R \ldots$' (both blanks to be filled the same way), so that '$Gy$' in the *definiens* is '$yRy$'. This, the simplest, course is adopted by Russell himself (*14.28).

(ii) One might take '$G$' to represent '$\ldots R\ (\iota x)(Fx)$', so that '$Gy$' in the *definiens* is '$yR\ (\iota x)(Fx)$'. The definite description '$(\iota x)(Fx)$' will then occur in the *definiens* with the scope '$yR\ (\iota x)(Fx)$', and must be eliminated by a second application of the definition. To avoid confusion of variables, we must introduce a new variable, '$z$' say, into the *definiens* of '$G\ (\iota x)(Fx)$', in place of '$y$'; since the context '$G \ldots$' is '$yR \ldots$', '$Gz$' in this *definiens* will be '$yRz$'.

(iii) We get similar results if we take '$G \ldots$' to represent '$(\iota x)(Fx)\ R \ldots$', so that '$Gy$' in the *definiens* is '$(\iota x)(Fx)\ Ry$'. To eliminate this occurrence of '$(\iota x)(Fx)$' with the scope '$(\iota x)(Fx)\ Ry$', we replace the variable '$y$' in the *definiens* of '$G(\iota x)(Fx)$' by '$z$', as before, and then take '$G \ldots$' to be '$\ldots Ry$', so that '$Gz$' in this *definiens* is '$zRy$'.

Now it is easy to prove that the results of these three ways of expounding '$(\iota x)(Fx)\ R\ (\iota x)(Fx)$' are logically equivalent. But they are not the same expression; and '$(\iota x)(Fx)\ R\ (\iota x)(Fx)$' cannot legitimately be used as an abbreviation for all three indifferently, until their logical equivalence *has* been proved; just as we cannot legitimately write '$a+b+c$' for '$(a+b)+c$' and '$a+(b+c)$' before establishing the associative law for addition.

I am sure a little ingenuity would enable one to find other cases, unnoticed by Russell and Whitehead, in which the same *definiendum*, containing a definite description, has more than one *definiens*; even, it may be, cases where the *definientia* are *not* logically equivalent.[1] Now complications like these cannot be allowed in good symbolism; so Russell's "contextual definition" of definite descriptions will not work as a symbolic device. (Various alternative conventions have been proposed, *e.g.*

[1] Such a proof that the conventions of "scope" are inconsistent has been put forward by Chwistek. *Cf.* M. Black, *The Nature of Mathematics*, p. 83.

Frege's and Quine's ; but it is not my concern to discuss these here).

An added defect of Russell's theory is that he defines the meaning of '$(\iota x)(Fx)$' in the existential assertion '$E!(\iota x)(Fx)$' by a special definition *14.02" and not in terms of the general definition of '$G(\iota x)(Fx)$'. His pretext is that it would be impossible to find a function to define '$E!y$', in the *definiens* '$(Ey):E!y.(x).Fx \equiv x=y$' that we should get by applying the general definition. But this is not true ; it is very easy to find a suitable function—e.g. '$(Ez)y=z$' (*cf.* *14.204). Moreover, without finding such a function, one could still bring '$E!(\iota x)(Fx)$' under the general definition, by defining it to mean '$F(\iota x)(Fx)$' (*cf.* *14.22).

*Cambridge, England.*

## ENTAILMENT AND THE MEANING OF WORDS

*By* S. Körner

THE purpose of this note is to consider two types of rules which assign labels to things and their relation to class-inclusions and entailments.

### I

Rules assigning labels to things are of many different kinds and to be able to distinguish between some of them is of philosophical importance. It is of great interest to inquire, for instance, into the different ways in which the labels 'green', 'chair', 'good', are assigned. At the very beginning of such an inquiry we might feel compelled to question the appropriateness of the phrase 'assigning *labels* to *things*' ; but we can for our limited purpose disregard this and other more subtle points and assume that there is no difficulty in identifying the thing to which a label is or is not to be assigned.

One main distinction is between rules of definite reference and rules of indefinite reference. *Rules of definite reference* assign the labels which they assign to each member of a finite class of things or to each member of a class of things the number of which is extendable *ad libitum*. They delimit what mathematicians and logicians call a 'class' and what 'I' shall call a 'definite class'. We recall Cantor's famous definition : "An aggregate

(class) is a collection into one whole of certain definite, well distinguished objects of our intuition or our thought . . . " (quoted, *e.g.*, by A. Fraenkel, *Einführung in die Mengenlehre*, Berlin 1928, New York, 1946). Subsequent attempts to improve on this definition preserve the requirement of definiteness. An example of a rule of definite reference is "Every integer derived by adding 2 to 0 any number of times to be called 'even'". *Rules of indefinite reference* assign labels by exemplification and cannot be replaced by rules of definite reference. The following sentence accompanied by appropriate pointing gestures is an example of a rule of indefinite reference: "This and this and this and everything like this to be called 'white'". I shall say that somebody is using a rule of reference if he is assigning labels in accordance with it.

Apart from rules of reference I shall consider classes and class-relations, *i.e.* unilateral and bilateral class-inclusions, class-overlaps and class-exclusions. I shall say that somebody is using a class-relation if he is asserting of a thing that it is, or is not, a member of the so related classes.

## II

I now consider rules of definite reference by means of a primitive example. Assume that the labels, '$a_1$', '$a_2$', '$a_3$', are assigned to three things and that each of them has one label assigned to it. A rule of reference which assigns at most one label to any one thing, I shall call a 'simple rule of reference'. The rule of the above example is a simple rule of definite reference. Another such rule is the rule which assigns the label '$b_1$', to the referent of '$a_1$'. By the conjunction of the two simple rules is formed the compound rule which assigns '$a_1$' and '$b_1$' to one thing, '$a_2$' to the second and '$a_3$' to the third. This rule is an example of what, I shall call an 'inclusive rule'; an inclusive rule being a compound rule of reference which assigns two sets of labels to a class of things in such a way that each thing has one label from the first set assigned to it and at most two labels altogether.

To every compound rule of definite reference there corresponds a class-relation between two classes. To use a compound rule of definite reference is necessarily to use a class-relation between the classes of the labelled things. The correspondence is many-one. The class-relation which corresponds to the above mentioned rule, corresponds also to any rule of reference which differs from it only in that instead of mentioning the labels '$a_1$', '$a_2$', '$a_3$', '$b_1$', it mentions, *e.g.*, '$c_1$', '$c_2$', '$c_3$', '$d_1$'.

The question whether class-relations can be used without the use of labels may be verbal, empirical, or both. If 'label' is defined narrowly, *e.g.*, admitting only words, there may be some people who can and some who cannot classify things without the use of labels. 'Label' may be defined in such a way that 'to classify without the use of labels' becomes a contradiction in terms, and if this is so it would be rather incongruous to declare as a profound discovery that things cannot be classified without using labels.

The compound inclusive rule of the above example stands in a many-one correspondence to the class-inclusion between the definite class of things labelled by '$a_1$', '$a_2$', and '$a_3$', and the definite class consisting of the thing labelled by '$a_1$', and '$b_1$'. It is generally agreed that some entailments and some class-inclusions are mutually replaceable. Thus with the help of suitable abbreviative definitions the assertion of our class-inclusion may be transformed into the assertion that 'being a b' entails 'being an a'. To be precise it would be necessary to indicate the contexts in which entailments and class-inclusions between definite classes are mutually replaceable and the purposes to which it makes no difference whether entailments or class-inclusions were used. There is a temptation to say loosely that the contexts in question are mainly situations in which people indicate the meanings of words for the purpose of using them to convey information of a different kind.

III

The rules which in the natural languages guide the use of most ordinary property- and relation-words are rules of indefinite reference. They are like the rule assigning the label 'white' which we have considered above and expressed with the help of the phrase 'everything like it'. A similar rule of reference assigns the label 'swan'. (I assume that the system of labelling under consideration does not contain the rule: "Only things which are to be labelled 'white' can be labelled 'swan'". Indeed this rule is neither a simple rule of reference nor a conjunction of such rules). It is easy, as already shown with simple rules of definite reference, to form a compound rule of indefinite reference by conjoining the rule assigning the label 'white' with the rule assigning the label 'swan'.

An interesting question follows: Is there, as with compound rules of definite reference, a many-one correspondence between compound rules of indefinite reference and class-relations,

especially class-inclusions? In particular, when the compound rule of the example is used, is a class inclusion also used? The answer depends on two considerations. Firstly, in order to say that simple rules of indefinite reference delimit classes and that such classes can stand in the relation of class-inclusion to each other, the meaning of 'class' must be extended to cover indefinite classes. This involves a serious deviation from common logical usage, as it violates the requirement of definiteness.

The second point is more important. A compound rule of *definite* reference corresponds, as already shown, in an obvious way to a class-inclusion or other class-relation. The conjunction of two simple rules of indefinite reference on the other hand does not correspond in the same way to a class-relation of the labelled things. If "This and this and everything like it to be labelled 'a'", and "that and that and everything like it to be labelled 'b'" are conjoined, then even if the classes of named things stand in the relation of class-inclusion, the indefinite classes whose elements are labelled 'a' and 'b' respectively, can still overlap. A class overlap and a class-inclusion between the indefinite classes, though incompatible with each other, are each compatible with the compound rule of indefinite reference. In other words, the use of a class-inclusion is not implicit in, but additional to the use of a compound rule of indefinite reference. It is not necessary to show in detail that some class-inclusions between indefinite classes and some entailments are mutually replaceable. Thus the indefinite class of all conceivable swans is included in the indefinite class of all conceivable white things if, and only if, 'being a swan' entails 'being white'.

## IV

So far then, the following results have been obtained. Some entailments and some class-inclusions between definite classes are replaceable by each other. Their use is implicit in the use of certain compound rules of definite reference (inclusive rules). Some entailments and some class-inclusions between indefinite classes are replaceable by each other. The use of these is not implicit in the use of any compound rule of indefinite reference. It follows, of course, that the decision to use a class-inclusion between indefinite classes or the corresponding entailment, is independent of any decision to use a compound rule of indefinite reference. It follows further that if it is proper to say that compound rules of reference determine the meaning of words, it is equally proper to say this of those entailments which supplement compound rules of indefinite reference.

Systems of labelling which contain only rules of definite and indefinite reference are a great deal simpler than most natural languages which include such systems and much besides them. Artificial languages such as have been constructed by Carnap and other logicians are in many respects more complicated than our systems of labelling. Thus the latter do not contain formation rules according to which labels of one kind can be combined into labels of other kinds, or according to which signs which are not labels are combined into labels. Yet in an important respect such systems of labelling are nearer to the natural languages than are the artificial systems of semantics. Unlike these and like the natural languages these systems contain rules of indefinite reference. I have shown that entailments have different functions in systems of definite reference and in systems of indefinite reference. Consequently any account of entailment which is based on a consideration of systems of definite reference only, cannot be adequate to systems containing rules of indefinite reference such as are the natural languages.

Let $L$ be a language containing rules of *indefinite* reference and class-inclusions between *indefinite* classes which are replaceable by entailments. It is then, as already shown, a mistake to say that the decision to use these entailments is implicit in the decision to use the rules of reference of $L$. On the other hand, if by, *e.g.*, ' the semantical rules of L ' is to be understood not only the rules of reference but also the entailments of $L$, then the decision to use the entailments of $L$ would indeed follow from the decision to use its semantical rules. This, however, would not be in any way surprising or illuminating.

*University of Bristol.*

# WHITEHEAD AND RUSSELL'S THEORY OF TYPES

*By* J. J. C. Smart

IN this short note I want to call attention to certain confusions which appear to be latent in Whitehead and Russell's symbolism and which consequently seem to vitiate their entire Theory of Types. Their symbolism, so long as we do not try to *interpret* it, certainly looks natural enough, for we are accustomed to write similar shapes when we are doing algebra. When, however, one does try to think of concrete examples one finds it more and more difficult to see how there could be a hierarchy of types in Whitehead and Russell's sense. The theory looks plausible only when it is left in its uninterpreted symbolism, and it looks plausible then simply because it contains patterns which look just like those we come across when doing mathematics.

We are quite familiar with the procedure of forming functions of functions in mathematics. Consider such functions as " $x^2$ ", " $x^2+1$ ", " $\sin x$ ". These are all functions of " $x$ ". Now suppose $f(x) = x^2+1$ and $\phi(x)=\sin x$. Then $\phi\{fx)\} = \sin(x^2+1)$ This is still a function of " $x$ ". That is, functions in mathematics, i.e. *descriptive* functions, are in a sense homogeneous. A function of a function is itself a function of what the second function is a function of, and we can go on building up functions of functions, and so on, in a way that is perfectly familiar to every schoolboy. That is, if " $f(x)$ ", " $\phi(x)$ ", " $\psi(x)$ " are descriptive functions, we feel quite at home with an expression like " $f[\phi\{\psi(x)\}]$ " and could easily give examples to illustrate its meaning, for example, " $\log \sin(x^2+1)$ ". " $\text{Log} \sin(x^2+1)$ " is a function of " $\sin(x^2+1)$ ", it is also a function of " $x^2+1$ ", and it is also a function of " $x$ ".

Contrast now *propositional* functions. Because there is a clear meaning to " $f[\phi\{\psi(x)\}]$ " in algebra we think that there must be a clear meaning to the same expression where " $f(x)$ ", " $\phi(x)$ " and " $\psi(x)$ " are *propositional* functions. Now this is just false.

Consider a function of a *propositional* function. What could we say of the propositional function " $\phi(x)$ " = " x is a man ", for example ? Well, one thing we might say (and this example will do just as well as any other), is that it has at least one true value. What is it that has at least one true value ? It is " $\phi(x)$ " not $\phi(x)$. Indeed " $\phi(x)$ has at least one true value " does not make sense. " x is a man has at least one true value " is not even grammatical, for it has two verbs. It is " x is a man "

that has at least one true value. Now consider the function " y is a true value of 'x is a man' ". This is a function of " x is a man " : it is in a sense a function of a function, but it is not a function of " $x$ ". " $x$ " occurs within the inverted commas. How different is the mathematical case, *e.g.* " log sin x ", where we have no interior inverted commas. Moreover when we put in the inverted commas we see that we are back on the first rung of the ladder again. " 'Socrates is a man' is of the subject predicate form " is about " Socrates is a man " and not about Socrates, just as " Socrates is a man " is about, not " Socrates ", but Socrates. Now surely it needs no theory of types to tell us that we cannot say things about functions that we can say about men. Men can be tall, but functions can't, just as functions can be of the subject predicate form while men can't. It no more needs the theory of types to tell us to watch our step in this regard than it does to tell us to sniff when someone says that virtue is blue or that the British Constitution has a red nose. What *would* be useful would be the injunction " watch your inverted commas ", and it is this warning that Whitehead and Russell have not heeded.

If we connect our " $\phi$ ", " $\psi$ ", etc., with *statement connectives* we can build up functions of functions in a way more like that with which we are familiar in algebra. There is a sense in which " $\phi x \vee \psi x$ " is a function of " $\phi$ " and " $\psi$ " and also of " $x$ ". There are no interior inverted commas in " $\phi x \vee \psi x$ " any more than in " log sin $x$ ". But " $\phi x \vee \psi x$ " is of the same type as " $\phi x$ " and " $\psi x$ " and Whitehead and Russell do not regard it as a function of a function in the required sense ; they agree that we are still on the first rung of the ladder. Here they are of course right.

My difficulty, then, is to see how one gets off the first rung of the ladder ; how one gets a hierarchy at all. Either one builds up with statement connectives, in which case our functions are still of the same type, or one says something *about* functions, *i.e.* one says something about " $\phi x$ " not about $\phi x$. (Strictly, the last clause, " not about $\phi x$ " is bad logical grammar, makes no sense). Our statement is of the form " F (' $\phi x$ '), *not* " F ($\phi x$) ".

Similar criticisms apply also to the ramified theory of types. Yet clothed in symbols it, too, looks natural. For " $(\phi).\ F(\phi, x)$ " reminds us of " $x \int_0^a y\,dy$ ", for example. The former is no more a function of " $\phi$ ", we are ready to be told, than the latter is of " y ". If we have done a little mathematics at school we

feel quite satisfied with Whitehead and Russell's symbolism. It is so like what we have been used to. It is all right so long as we don't *think* about it.

Consider the *Principia Mathematica* example: "Napoleon had all the qualities of a great general". Two remarks are immediately relevant. (1) The word "quality" should put us on our guard for concealed inverted commas, for the use of the material mode of speech. (2) Even in the material mode of speech language usually takes care of the relevant distinctions. It is *confidence* that is a quality of a great general, while the great general is *confident*. The *Principia Mathematica* symbolism does *not* take care of the distinction; there is the same symbol "$\phi$" in "$f(\phi)$" as in "$\phi$ (Napoleon)". (I do not here use the "capped notation"; perhaps it has something to do with the distinction I have in mind. At any rate we are not clearly *told* how it would help to write "$\phi\hat{x}$" instead of "$\phi$".)

Let us translate the sentence "Napoleon had all the qualities of a great general" into the formal mode of speech. We get something like the following:—

(W) (N): *If* N and W are put into the blanks of "— is —" and we get a true sentence, and if N is put into the blank of "— is a great general", and we get a true sentence, *then* if W is put into the blank of "Napoleon is —" we get a true sentence.

That is, our variables take *words* as their values. The statement is about W (or "$\phi$") *not* about $\phi$. Now a statement about "$\phi$" is just as much on the first rung of the ladder as one about Napoleon. The only thing, of course, is that one can't say things about words, nor yet about qualities, that one can about men. But here we need no technical theory of types to tell us this. Complete rigour can be achieved simply by avoiding the material mode of speech and by observing the rules for the use of inverted commas.

It is not type muddles (in Whitehead and Russell's sense) that we must guard against; it is inverted comma muddles. If we don't fall into inverted comma muddles we see that Whitehead and Russell's theory of types is both unintelligible and unnecessary.

Of course one motive for Whitehead and Russell's theory of types is the desire to evade the logical paradoxes. These, however, have nothing in particular to do with the 'vicious circle principle' and Whitehead and Russell have not found the right method for dealing with them. But that is another story and outside the scope of this note.

Finally, I must make it clear that I am not arguing against a theory of types, in some form or another, within mathematics. We must not, for example, confuse the real number 3 with the rational number 3. I am merely protesting against some features of the calculus of propositional functions.

*Corpus Christi College, Oxford.*

–
–
a
s.
e
s